Algorithm study: Josephus Problem

**The survivor label = (((((0+m1%2)+m2)%3+m3)%4 +m4)%5+m5)%6....**  
  
Examples:  
  
Player : [0..5]  
If m=3,  
{0,1,2,3,4,5}  
=>{0,1,3,4,5} - 2  
=>{0,1,3,4} - 5  
=>{0,1,4} - 3  
=>{0,1} - 4  
=>{0} - 1  
  Saved=(((((0+3)%2+3)%3+3)%4+3)%5+3)%6  
  = 1->1->0->3->0  
  
  
Player : [0..4]  
  
  
If m=3,  
{0,1,2,3,4}  
  
==>{0,1,3,4} -2  
  
==>{0,1,3} -4  
  
==>{1,3} - 0  
  
==>{3} - 1  
  
  Saved=((((0+3)%2+3)%3+3)%4+3)%5  
  = 1->1->0->3  
  
  
Player : [0..3]  
  
  
If m=3,  
{0,1,2,3}  
  
==>{0,1,3} -2  
  
==>{0,3} - 1  
==>{0} - 3  
  
  Saved=(((0+3)%2+3)%3+3)%4  
  = 1->1->0  
  
  
  
Player : [0..2]  
If m=3,  
{0,1,2}  
  
==>{0,1} -2  
  
==>{1} - 0  
  Saved=((0+3)%2+3)%3  
  = 1->1  
  
  
  
  
Player : [0..1]  
  
If m=3,  
{0,1}  
  
==>{0,1} -0  
  
==>{1}  
  Saved=(0+3)%2  
  = 1  
  
  
Player : [0]  
No need to play!  
Saved= 0  
  
But, Why?  
  
====================  
Proof.  
If there is n people, last killed guy is X, (X+m)%n will be the next guy to be killed.  
If there is n people, killed guy is Y, (Y-m)% (n) will be the prev guy to be killed.  
and all label after >= (Y-m)%(n) will be +1, then the last guy labelled as (Y-m)%n;  
  
Ok, we do the process backward.  
If the saviour guy is 0. add dummy 1.  
(0-m)%2 will be the prev guy to be kill => (0-3)%2 =-3%1=1.  
If we enforce prev guy to be zero, all other guys need to advance to position (?+m)%2,   ?={0,1}  
0->1  
Then we add this new prev guy as '0'  
1->0  
The total displacement of the surviour = 0+m%2 = 1.  
Result:{0,1}  
  
Then in next round, added a dummy 2.  
(0-m)%3 will be prev guy to be killed = -3%3=0  
If we let this guy to be zero, all guys need to advance to position (?+m)%3,  ?={0,1,2}  
1->1  
2->2 (dummy)  
Then we add this new guy as '0'.  
0->0;  
  
The total displacement of the surviour = ((0+m%2)+m)%3  
Result:{0,1,2}  
  
  
  
Then in next round, added a dummy 3.  
(0-m)%4 will be prev guy to be killed = -3%4=1  
If we let this guy to be zero, all guys need to advance to position (?+m)%4,  ?={0,1,2,3}  
0->3  
2->1  
3->2 (dummy)  
Then we add this new guy as '0'.  
1->0;  
  
The total displacement of the surviour = (((0+m%2)+m)%3+m)%4  
Result:{0,1,2,3}  
  
  
  
  
  
Then in next round, added a dummy 4.  
(0-m)%5 will be prev guy to be killed = -3%5=2  
If we let this guy to be zero, all guys need to advance to position (?+m)%5,  ?={0,1,2,3,4}  
0->3  
1->4  
3->1  
4->2 (dummy)  
Then we add this new guy as '0'.  
2->0;  
  
The total displacement of the surviour = ((((0+m%2)+m)%3+m)%4+m)%5  
Result:{0,1,2,3,4}  
  
  
  
Then in next round, added a dummy 5.  
(0-m)%6 will be prev guy to be killed = -3%6=3  
If we let this guy to be zero, all guys need to advance to position (?+m)%6,  ?={0,1,2,3,4,5}  
0->3  
1->4  
2->5  
4->1  
5->2(dummy)  
Then we add this new guy as '0'.  
3->0;  
  
The total displacement of the surviour = (((((0+m%2)+m)%3+m)%4 +m)%5+m)%6  
Result:{0,1,2,3,4,5}  
  
\* If m is changing in each round, the equation will be:  
  
The total displacement of the surviour = (((((0+m1%2)+m2)%3+m3)%4 +m4)%5+m5)%6.....